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PHASE VELOCITIES AND DISCONTINUOUS STRUCTURE OF SHOCK WAVES

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§1. The problem of the structure of a shock wave consists in the search for the solution $u_k(x, t) = u_k(x - Ut)$ $(k = 1, \ldots, n)$ of a system of quasilinear equations of the form

$$\frac{\partial}{\partial t} A_i(\mathbf{u}) + \frac{\partial}{\partial x}\left[B_i(\mathbf{u}) - \sum_{k=1}^{m} \alpha_k C_{ik}(\mathbf{u}) \frac{\partial u_k}{\partial x}\right] = 0, \quad i = 1, \ldots, m, \tag{1.1}$$

$$\frac{\partial}{\partial t} A_i(\mathbf{u}) + \frac{\partial}{\partial x} B_i(\mathbf{u}) = 0, \quad i = m+1, \ldots, n,$$

with the boundary conditions $du_k/dx|_{x=\pm\infty} = 0$, where $\mathbf{u} = \{u_k\}_1^n$ is the set of parameters characterizing the state of the medium and $\{\alpha_k\}_1^m$ are the dissipative coefficients.

The extreme complexity of this problem, which arises largely due to the possibility of existence of segments of irregular behavior of the solution, does not permit the solution in its general formulation. At the same time, the determination and elimination of the irregular segments can appreciably simplify the problem. Such irregularities appear in the form of nonphysical segments in the solution, which generally correspond to regions of multivaluedness of some functions $u_k(x)$. The nonphysical segment in a formal mathematical solution must be replaced by a discontinuity, as usually done in hydrodynamics or magnetohydrodynamics in the study of shock waves [1-5]. It has been noted in a number of studies that an internal discontinuity appears in the case when the flow velocity in the wave goes through a certain critical value [2, 6-9]. Furthermore, it is shown that the critical velocity is the phase

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velocity of the highest ideal system (Whitham–Lyubarskii criterion) [8-10]. Subsequently, it was shown that the transition through the critical velocity does not always result in the development of the discontinuity [5, 11]. Below we discuss the role of the phase velocities of ideal systems in the formation of a discontinuity within a shock-wave profile.

§2. We call the system of equations (1.1) a dissipative system of order m. Dissipative systems of lower order can be obtained from system (1.1) by putting one or a number of dissipative coefficients equal to zero (see, for example, (2.3) [12]).

The discontinuity in the solution of a dissipative system of order r (if such a discontinuity appears) will be called a discontinuity of order r. With each dissipative system of order r there can be an associated ideal system if all the nonzero dissipative coefficients are made to tend to infinity [10, 12]:

$$\sum_{k=1}^{r} C_{ik}(\mathbf{u})\,\partial u_k/\partial x = 0,\quad i = 1,\ldots,r,$$
$$\frac{\partial}{\partial t}A_i(\mathbf{u}) + \frac{\partial}{\partial x}B_i(\mathbf{u}) = 0,\quad i = r+1,\ldots,n, \tag{2.1}$$

which admits of a solution in the form of plane waves with nondamping amplitude. The characteristic equation of system (2.1)

$$D_r(\mathbf{u}, U) \equiv \left|\frac{\partial}{\partial u_k}B_i(\mathbf{u}) - U\frac{\partial}{\partial u_k}A_i(\mathbf{u})\right|_{r+1}^{n} = 0,\quad 0 \leqslant r \leqslant m, \tag{2.2}$$

determines n − r real phase velocities $U = V_r^j$, j = 1, ..., n − r [8, 10, 12]. The unimportant factor $|C_{ik}|_1^r$, which cannot vanish by virtue of the dissipative nature of system (1.1) [10], is omitted in (2.2). The phase velocities of an ideal system of order r will be called phase velocities of order r for brevity.

In a reference system moving with the shock wave (U = 0) the dissipative system of equations takes a simpler form:

$$\alpha_i \frac{d}{dx}u_i = b_i(\mathbf{u}),\quad i = 1,\ldots,m,$$
$$b_i(\mathbf{u}) = 0,\quad i = m+1,\ldots,n. \tag{2.3}$$

The boundary conditions are now written in the following way: $u_k(-\infty) = u_k^-$, $u_k(+\infty) = u_k^+$. The values $u_k^\mp$ (k = 1, ..., n) are the solutions of the lowest ideal system $b_i(\mathbf{u}) = 0$ (i = 1, ..., n) corresponding to the absence of dissipation in the medium (m = 0).

The determinants of all ideal systems associated with dissipative system (2.3) are obtained from the determinant of the lowest ideal system $D_0(\mathbf{u}) = |b_{ik}(\mathbf{u})|_1^n$, $b_{ik}(\mathbf{u}) \equiv (\partial/\partial u_k)\cdot b_i(\mathbf{u})$ by deleting the rows and columns with the numbers of the dissipative coefficients that tend to infinity. It is obvious that the determinant D_r of the ideal system changes sign if the flow velocity within the shock layer crosses the corresponding phase velocity.

According to the Whitham–Lyubarskii criterion the development of the discontinuity in the shock-wave profile is related to the transition of the flow velocity v_x within the shock layer through the highest-order phase velocity,

$$v_x^- > V_m > v_x^+,$$

or to the change of sign of the determinant of the highest-order ideal system [8, 10],

$$D_m(\mathbf{u}^+)D_m(\mathbf{u}^-) < 0. \tag{2.4}$$

As already mentioned, in spite of the need for this criterion [5, 8], it is found to be inadequate in the general case. This inadequacy arises due to the possibility of occurrence of singular points (different from the boundary singular points $\mathbf{u}^-$ and $\mathbf{u}^+$), through which the solution may pass in such a way that the discontinuity does not arise [5, 11]. However, under certain restrictions the problem of development of a discontinuity in the solution of a dissipative system of an arbitrary order can reduce to the determination of a sequence of discontinuities similar to the first-order discontinuity, i.e., to the solution of a sequence of problems without internal singular points. It turns out that the phase velocities of all orders play a significant role in the formation of the discontinuity.

§3. In order to simplify the subsequent discussion, we shall restrict ourselves to the following relationships among the dissipative coefficients:

$$\alpha_1 \gg \alpha_2 \gg \ldots \gg \alpha_m. \tag{3.1}$$

This choice of the relationships permits one to consider in each order r ($0 \leqslant r \leqslant m$), C_m^r for only one ideal system corresponding to the first r dissipative coefficients tending to infinity and the remaining $(m - r)$ coefficients tending to zero instead of a set of C_m^r. Furthermore, in order to avoid investigating the evolution of the emerging discontinuities, we shall assume the wave intensity to be such that the determinants $D_r(\mathbf{u})$ ($0 \leqslant r \leqslant m$) do not change sign more than once on the interval $(\mathbf{u}^-, \mathbf{u}^+)$.

Along with dissipative system (2.3), dissipative systems of order $(m - 1)$ to the first, which can be obtained from (2.3) depending on the degree of idealization of the problem by successive disregarding of the dissipative processes governed by the coefficients $\alpha_m \ll \alpha_{m-1} \ll \ldots \ll \alpha_2$, are also meaningful.

We shall assume that the solution of the dissipative system of order m has a discontinuity, i.e., condition (2.4) is satisfied. Then in view of the fact that the elimination of any dissipative process may not remove the discontinuity, the solution of the dissipative system of order $m - 1$ ($\alpha_m = 0$) also must have a discontinuity, i.e., according to the Whitham–Lyubarskii criterion, the condition $D_{m-1}(\mathbf{u}^+)D_{m-1}(\mathbf{u}^-) < 0$ must be satisfied. Next, passing on to the system of order $m - 2$ ($\alpha_m = \alpha_{m-1} = 0$), we obtain the condition $D_{m-2}(\mathbf{u}^+)D_{m-2}(\mathbf{u}^-) < 0$. Continuing this argument, we arrive at the conclusion that a necessary condition for the existence of a discontinuity of order m within a shock-wave profile is the transition through the phase velocities of ideal systems of all orders from order zero to m inclusive, i.e., at least the relationships $D_r(\mathbf{u}^+)D_r(\mathbf{u}^-) < 0$, $r = 0, 1, \ldots, m$, must be satisfied.

§4. In the absence of dissipation, the shock wave can not otherwise be described as a discontinuity that connects the constant values of the gas parameters u_k^- ahead of the wave and u_k^+ behind it [1, 13, 14]. Besides, in the stationary case the unperturbed flow may form a boundary only with the shock discontinuity [15], i.e., the relation

$$D_0(\mathbf{u}^+)D_0(\mathbf{u}^-) < 0 \tag{4.1}$$

must be satisfied for $\mathbf{u}^- \neq \mathbf{u}^+$.

The introduction of dissipation brings in a certain length scale to the problem. Actually, the solution $\mathbf{p}(x)$ of the dissipative system of first order

$$\alpha_1 dp_1/dx = b_1(\mathbf{p}),\ b_i(\mathbf{p}) = 0,\ i = 2, \ldots, n, \tag{4.2}$$

describes a continuous variation of the parameters of the medium $p_k(x)$ from the values u_k^- to u_k^+, and this variation occurs over a length of the order of α_1 [1, 13, 16]. The solution of system (4.2) remains continuous only as long as the condition $D_1(\mathbf{u}^+)D_1(\mathbf{u}^-) > 0$ is satisfied. If the Whitham–Lyubarskii criterion

$$D_1(\mathbf{u}^+)D_1(\mathbf{u}^-) < 0, \tag{4.3}$$

holds in the interval $(\mathbf{u}^-, \mathbf{u}^+)$, then a first-order discontinuity develops within the shock-wave profile [5, 8, 10], i.e., the functions $p_k(x)$ ($k = 2, \ldots, n$) change their value with a jump from $p_k(1) = p_k(x_1 - 0)$ to $p_k(2) = p_k(x_1 + 0)$. Here x_1 indicates the position of the internal discontinuity. As regards the function $p_1(x)$, it follows from the first equation of system (4.2) that the derivative dp_1/dx is always finite and $p_1(x)$ itself is continuous:

$$p_1(1) = p_1(2) = p_1(x_1). \tag{4.4}$$

It must be noted that if condition (4.1) is not satisfied, then system (4.2) has only the trivial continuous solution $p_k(x) = \text{const}$ ($k = 1, \ldots, n$), i.e., the first-order discontinuity may be contained only in the zero-order discontinuity smeared by dissipation.

As a rule, the solution of first-order dissipative systems does not present essential difficulties due to the absence of any internal singular points. The direct solution of such systems in ordinary hydrodynamics and magnetohydrodynamics shows [1, 2, 8, 13, 14] that the first-order discontinuity arises as a result of overturning of the shock-wave profile. The fact that the overturning occurs during the transition through the corresponding phase velocity within the shock-wave profile (the phase velocity of isothermal [1] and isomagnetic oscillations [8], etc.) shows that the mechanism of formation of the internal discontinuity is similar to the mechanism of formation of the shock wave (zero-order discontinuity).

§5. We consider a second-order dissipative system:

$$\alpha_1 dq_1/dx = b_1(\mathbf{q}),\ \alpha_2 dq_2/dx = b_2(\mathbf{q}), b_i(\mathbf{q}) = 0,\ i = 3, \ldots, n. \tag{5.1}$$

For $\alpha_2 = 0$, system (5.1) goes over into (4.2), and the functions $q_k(x)$ must go over into $p_k(x)$. It is also clear that as $\alpha_2 \to 0$ the left-hand side of the second equation of (5.1) can be disregarded only in the case when the derivative $dq_2/dx \to dp_2/dx$ does not take on large values. In other words, the term $\alpha_2 dq_2/dx$ is significant and the solution $\mathbf{q}(x)$ is qualitatively different from $\mathbf{p}(x)$ only in a small region (of the order of α_2), within which $p_2(x)$ undergoes discontinuity. Outside this region, and also in the case when the solution $\mathbf{p}(x)$ of the first-order system does not contain a discontinuity, the solution $\mathbf{q}(x)$ of the second-order system has the same nature as $\mathbf{p}(x)$ [11], and, in principle, it can be found by the perturbation method [16].

Since the problem is not to find the exact solution of the dissipative systems but to investigate its singularities, on the basis of the above discussion it may be asserted that the "turning on" of the dissipative process with $\alpha_2 \ll \alpha_1$ leads, in practice, only to a smearing of the first-order internal discontinuity in the region of continuous variation of the parameters with width α_2, i.e., it can be approximately assumed that outside this region the solution $\mathbf{q}(x) = \{q_k(x)\}_1^n$ of dissipative system (5.1) coincides with the solution $\mathbf{p}(x) = \{p_k(x)\}_1^n$ of system (4.2). By virtue of (4.4), the function $p_1(x)$ may be assumed to coincide with $q_1(x)$ in the entire range of variation of x.

Since the second-order transition region $x_1 - \alpha_2 < x < x_1 + \alpha_2$ is small compared to the width of the first-order transition region $x_1 - \alpha_1 < x < x_1 + \alpha_1$, inside this region system (5.1) can be replaced by the first-order system

$$q_1(x) = p_1(x).\ \alpha_2 dq_2/dx = b_2(\mathbf{q}),\ b_i(\mathbf{q}) = 0,\ i = 3, \ldots, n, \tag{5.2}$$

and the boundary conditions

$$q_k(x_1 - \alpha_2) = p_k(1),\ q_k(x_1 + \alpha_2) = p_k(2),\ k = 2, \ldots, n. \tag{5.3}$$

It is obvious that the system (5.2) thus obtained has the same characteristics as system (4.2): 1) the solution $q_k(x)$ $(k = 2, \ldots, n)$ contains a discontinuity if the Whitham–Lyubarskii criterion for system (4.2)

$$D_2(\mathbf{p}(1))D_2(\mathbf{p}(2)) < 0; \tag{5.4}$$

is satisfied; 2) it has only the trivial solution $q_k(x) = \text{const}$ if $\mathbf{p}(1) \equiv \mathbf{p}(2)$, i.e., if condition (4.3) is not satisfied. In other words, the second-order discontinuity can appear only within the first-order discontinuity smeared by dissipation.

Using similar arguments, the solution of a dissipative system of any order r can be constructed if the solution of the dissipative system of order $r - 1$ is known; it can be shown that the discontinuity of order r can appear only within the discontinuity of order $r - 1$ smeared by dissipation; here the condition

$$D_r(\mathbf{u}(1))D_r(\mathbf{u}(2)) < 0 \tag{5.5}$$

must be satisfied, where $\mathbf{u}(1)$ and $\mathbf{u}(2)$ are the values of the parameters of the medium on the two sides of the discontinuity of order $r - 1$ and $\mathbf{u}(1) \neq \mathbf{u}(2)$.

It is clear that condition (5.5) must be satisfied in all orders from zero up to and including m for the existence of a discontinuity of order m in the solution of system (1.1).

This result explains the absence of an isothermal, isomagnetic jump within the profile of a slow MHD shock wave of small intensity propagating in a nonviscous gas (m = 2) even under the condition when within its profile the flow velocity crosses the isothermal sound velocity (which, in this case, happens to be the phase velocity V_2 of the highest ideal system) [11]. Actually, in this case, the zero-order phase velocity V_0 is the slow magnetoacoustic velocity and the first-order phase velocity V_1 is the sound velocity (adiabatic). In view of the fact that the difference $V_1 - V_2$ always remains finite, with the Whitham–Lyubarskii criterion being satisfied in a slow MHD shock wave of small intensity (i.e., $|V_0 - V_2|$ has small values), the condition of existence of the first-order discontinuity is not satisfied and as a result the second-order discontinuity does not occur [11].

However, the situation changes significantly with the increase in the intensity of the slow shock wave. Thus, when the thermal conductivity of the medium is small compared to the magnetic viscosity, the slow MHD shock wave starts as an ordinary hydrodynamic wave (isomagnetic discontinuity, first-order discontinuity) if the flow velocity ahead of it exceeds the sound velocity [9]. An isothermal discontinuity which is also an isomagnetic isothermal discontinuity in the profile of the slow MHD shock wave may appear within an ordinary shock wave [5].

§6. The first-order discontinuity, if it exists, lies at the beginning or the end of the shock-wave profile. This has been established from the solution of the problem of the structure of a shock wave in ordinary gasdynamics [1] and in magnetogasdynamics [8] and is easily extended to the case of an arbitrary dissipative system of the first order. The shock-wave profile begins (ends) with the discontinuity if $V_1 > V_0$ ($V_1 < V_0$).

It has been shown above that the solution of dissipative system (2.3) can be reduced to the solution of a number of first-order dissipative systems. Making use of this result, it may be asserted that the discontinuity of order $r (0 \leqslant r \leqslant m)$ lies at the beginning of the discontinuity of order $r' = r - 1$ smeared by dissipation α_r if $V_r > V_{r'}$ and at its end if $V_r < V_{r'}$. The jumps of the functions at this discontinuity are determined by the boundary conditions of type (5.3):

$$\begin{gathered} \mathbf{u}(1r) = \mathbf{u}(1r'), \\ u_k(2r) = u_k(2r'),\ k = 1, \ldots, r, \\ b_i(\mathbf{u}(2r)) = 0,\ i = r + 1, \ldots, n \end{gathered} \tag{6.1}$$

for $V_r > V_{r'}$;

$$\begin{gathered} u_k(1r) = u_k(2r),\ k = 1, \ldots, r, \\ b_i(\mathbf{u}(1r)) = 0,\ i = r + 1, \ldots, n, \\ \mathbf{u}(2r) = \mathbf{u}(2r') \end{gathered} \tag{6.2}$$

for $V_r < V_{r'}$; here (1r) and (1r') denote the states ahead of discontinuities of order r and $r' = r - 1$; (2r) and (2r') denote the states behind them.

Thus, not only the presence of the discontinuity can be ascertained, but also its magnitude and position within the shock-wave profile can be found without solving dissipative system (1.1).

§7. We note two particular cases in which the fact of existence of the discontinuity, its magnitude, and its position are determined uniquely independently of the relationships among the dissipative coefficients. Actually, if the relationships $V_0 < V_1 < \ldots < V_r < \ldots < V_m$ hold for any order of inclusion of the dissipative coefficients, then the transition through the phase velocity of the highest ideal system V_m automatically denotes satisfying conditions (5.4) at all orders from zero up to and including $m - 1$, and independently of the order of inclusion of the dissipative coefficients, condition (6.1) also reduces to

$$\begin{gathered} \mathbf{u}(1m) = \mathbf{u}^-, \\ u_k(2m) = u_k^-,\quad k = 1, \ldots, m, \\ b_i(\mathbf{u}(2m)) = 0,\ i = m + 1, \ldots, n, \end{gathered} \tag{7.1}$$

i.e., the shock wave starts as a discontinuity of order m and the jumps of the functions can be obtained from conditions (7.1).

On the other hand, if the inverse relationships $V_0 > V_1 > \ldots > V_r > \ldots > V_m$ hold among the phase velocities of all ideal systems for any order of inclusion of the dissipative coefficients, then the transition through the phase velocity of the highest ideal system V_m also automatically results in conditions (5.4) being satisfied at all orders. In this case the shock wave ends with a discontinuity of order m and the jumps of the functions can be obtained from the conditions

$$\begin{gathered} \mathbf{u}(2m) = \mathbf{u}^+, \\ u_k(1m) = u_k^+,\quad k = 1, \ldots, m, \\ b_i(\mathbf{u}(1m)) = 0,\ i = m + 1, \ldots, n, \end{gathered}$$

to which conditions (6.2) reduce independently of the order of inclusion of the dissipative coefficients.

As an example, one could consider a fast MHD shock wave propagating in a nonviscous medium with finite thermal and electrical conductivities ($m = 2$), in which (in contrast to the slow shock wave) the transition of the flow velocity through the isothermal sound velocity leads to the appearance of an isothermal, isomagnetic discontinuity [4].

§8. It was noted above that [in approximation (3.1)] the procedure of search for a discontinuity in the solution of a dissipative system could be reduced to the search for a system of first-order discontinuities accompanying successive "freezing" of the degree of freedom, which would permit one to discard one of the differential equations of the system at each step, thus making the procedure converge.

It is quite clear that such "freezing" of the value of, for example, the parameter u_r is equivalent to making the dissipative coefficient α_r go to infinity, i.e., at each step of the procedure one passes on to the ideal system corresponding to the nonzero (but finite according to the assumption) dissipative coefficient tending to infinity. Here the discontinuity is defined as a shock wave in the ideal medium.

The paradox of the situation is related to the fact that the equilibrium structure of a steady-state shock wave has been considered. However, this paradox can be eliminated if the dynamics of setting up of the steady-state profile is taken into consideration. This can be done most vividly in the case of a shock wave developing in a thermally conducting gas.

In a reference system attached to the shock wave, the existence of a stationary structure of the front means that in each cross section of the front there exists an equilibrium between the heat flux carried by the gas flowing across this section and the heat flux in the opposite direction caused by the temperature gradient and the finite conductivity of the gas.

It is well known that the existence of nonhydrodynamic heat transfer in the leading layers of the front leads to the result that in the process of formation of the stationary shock wave the temperature in the wave changes more rapidly than the other parameters. Thus, when the intensity of the shock wave under formation increases, it is found that in some cross section of the as yet nonequilibrium front the temperature reaches the value T^+ established behind the shock front. It is obvious that a further increase of the temperature in this cross section, as also in all layers behind it, is impossible [13].

Actually, the existence of a region within the wave profile where the temperature gradient is directed into the region behind the wave would mean the opposite. This would indicate that the heat flux carried by the gas passing through the investigated zone will not be compensated forby the heat flux arising due to thermal conductivity, but will be added to it, which would lead to an instantaneous decrease of the temperature to T^+. At the same time, any temperature increase behind the front would rapidly lead to an increase of the temperature within this zone.

Thus, it is evident that for sufficiently large intensity of the wave in a medium with finite (small) thermal conductivity an isothermal region may exist within the wave where the gas behaves as an ideal medium with infinitely large effective thermal conductivity. In this medium the isothermal oscillations are undamped and the velocity of propagation of isothermal perturbations is smaller at the beginning of the isothermal zone than at the end. Due to the absence of dissipation in this zone, conditions for the buildup of the perturbations at the discontinuity appear.

We shall restrict ourselves to the example cited above, since similar arguments can be presented in the case of any first-order dissipative system.

Since isovelocity discontinuities (tangential, Alfvén) cannot belong to a shock wave, the phase velocities of ideal systems corresponding to dissipative systems with nonzero viscosity coefficients must be special [12].

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RADIATION ORIGINATING BY THE IMPACT OF A GAS LAYER AGAINST AN OBSTACLE AT VERY HIGH VELOCITIES

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For some time now, various devices that allow gases to be accelerated to very high velocities have been constructed. As an example, we point out the erosion-type magnetoplasma compressors [1-5], in which maximum velocities (70-90 km/sec) with a quite high density of the gas jet are being successfully achieved. Deceleration occurs when this jet impacts against an obstacle and the kinetic energy of the gas is converted into internal energy. As the temperature of the heated gas becomes high, the emission of the plasma can be considerable. This effect has already been used in experiments [4, 5] in order to increase the energy conversion factor of an electric battery, feeding a plasmodynamic discharge, into radiation energy. It will be of theoretical interest to estimate the principle characteristics of the heated gas and the resulting radiation pulse for different jet parameters (velocity, density, and length) which could then be used to find their optimum values.

The pattern of the motion and transfer of radiation in the case of an arbitrarily shaped obstacle and with an arbitrary distribution of the parameters in the jet can be extremely complex, and for its description the time-consuming solution of the two-dimensional nonsteady radiation-gasdynamic problem is necessary. However, in certain cases, this phenomenon can proceed under conditions which are quite close to one-dimensional plane geometry (for example, if the jet impacts on the plane base of an evacuated cylindrical "bucket," as if "cutting out" of it a uniform central part, which occurred in [4], and the times being considered are such that the resulting shock wave traverses a distance which is less than the diameter of the bucket).

We shall carry out some estimates of the parameters of a plasma heated up by the impact against an obstacle. Suppose that the average velocity of the jet is ~50 km/sec and the

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